Examiners' Report

Principal Examiner Feedback

November 2021

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 02

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Introduction

This was an additional sitting of this paper due to the extraordinary circumstances we are in at the present time.

Centres should encourage their students to read questions very carefully including the mark allocation as this gives a good indication of the amount of work involved. In particular, the instruction 'write down' involves minimal work and can mostly be answered by inspection. The instruction 'show that' on the other hand means that we need to see every step in working in order to be able to award the available marks. It is better to add in an extra step rather than leave one out as it is unfortunately sometimes the case.

Question 1

Part (a) On the whole, this was a well-answered question for approximately half of the candidates.

As the formula for $\cos(A+B)$ is given on the formulae sheet, it was disappointing to see so many poor attempts to start this question, in some rare cases candidates did not even start this part of the question at all.

Generally, almost all attempted the question however and those that applied the formula given had clearly learnt the required identity of $\cos^2 A+\sin^2 A=1$ in order to complete the required proof and gain the two marks available.

There were a number of candidates who did not show the proof in full; these went straight from the formula for $\cos(A+B)$ to $\cos 2A=\cos^2 A-\sin^2 A$ and then directly to the given result. This is a 'show that' question and without seeing the intermediate step of changing $\cos^2 A$ to $1-\sin^2 A$, there is no way of knowing if this fact is known or simply copied from the given result, therefore for the M mark, candidates had to show that they had made the substitution of $\cos^2 A=1-\sin^2 A$. It also goes without saying that any errors in this 'show that' question resulted in the final A mark being lost.

Part (b) The first M mark was achieved by most candidates. To gain this mark, they had to correctly substitute the given results for $\sin A$ and $\cos A$ into the identity in (a). Some candidates choose to attempt to find the algebraic expansion for $\sin^2 A$ first before making this substitution. There were some errors seen in the expansion of $\left(\frac{x+1}{2}\right)^2$, which for candidates sitting this paper is disappointing to see. The most common error was a failure to square the denominator.

There were occasions where candidate did try to rearrange the equation to make y the subject before substitution of $\sin^2 A$ but this generally led to errors in rearranging.

The second M mark was awarded for reaching an expression for y in terms of x. Some candidates stopped when they could see they were not going to get to the required result, but not reaching to any form of $y = \mathrm{f}(x)$ meant that they could not score this second M mark.

A good number of candidates were able to manipulate the algebra correctly and achieve the required result. Those that did not gain all the marks usually made sign errors when removing the bracket from their expansion of $\left(\frac{x+1}{2}\right)^2$, the negative sign in-front of the fraction was incorrectly used on some of the terms in the numerator, resulting in the incorrect final result. The most common example of this mistake was;

$$1-2\left(\frac{x^2+2x+1}{4}\right)=1-\frac{1}{2}x^2+x+\frac{1}{2}$$

where the negative sign was not correctly used.

Question 2

The first section of the question seems fairly standard and straightforward. However, some candidates did not pick up M1A1 for the following reasons:

- Aa attempt to solve $4-x^2=0$ and/or $x+2=0$
- No attempt whatsoever

- Forming $4-x^2=x+2$ but algebraic slips resulted in an incorrect quadratic that was/was not solved in an acceptable manner. Centres should note that it is always advisable to show a full method for solving a quadratic equation, as incorrect roots from an incorrect quadratic without showing a valid method will result in M0.

The most common marking pattern for the remaining 4 marks resulted in M0M1M1A0.

The majority of candidates did not state the correct form for the volume of rotation. The common incorrect answers were integrating the result of the curve equation minus the line equation or subtracting two equations first, then squaring them resulting in $\pi\int_{-2}^{1}\left(x^2+x-2\right)^2\,\mathrm{d}x$ This inevitably led to losing the first M mark and therefore also the last A mark.

Integration of candidates' expressions (even if the expression for the volume was incorrect) was usually accurate and most satisfied the criteria for an attempt at integration to score the M mark. Candidates were less efficient at showing an explicit substitution of their limits in their integral. Where explicit substitution of their limits was not shown an incorrect final answer would not be able to gain the 3rd M mark. Once again, centres should impress upon their students that full working must always be shown.

Candidates who attempted finding the volume of the cone by using $\frac{1}{3}\pi r^2 h$ were usually more successful than those who attempted to find the volume purely by integration (Method 1 in the scheme). However, a few candidates were seen adding the volume of the curve rotation with the volume of the cone rather than subtracting, which led to losing both 2nd M and final A marks, even though all their previous work was perfectly correct.

A very small minority of candidates were seen using the volume of the line rotation minus the volume of the curve rotation, reaching the volume of $-\frac{108}{5}\pi$ and having realised that the volume should be negative, hence removed the negative sign when giving the conclusion of the volume. This was not penalised and scored all appropriate marks.

Question 3

This question was answered poorly by many candidates who find this topic difficult and have no concept of finding asymptotes and find reciprocal graphs generally difficult to deal with.

Part (a) was answered by almost half of the candidates. The instruction here was 'write down' and as there were only 2 marks available, for values of *c* and *d*, which implies a minimal amount of work, but some candidates wrote half a page of work. The denominator in the equation of the curve was $(cx-d)$ and the equation of the vertical asymptote was $x=\frac{3}{2}$ meaning that $\frac{c}{d}=\frac{3}{2} \Rightarrow c=3 \quad d=2$ because the question states that *a*, *c* and *d* are prime numbers.

Part (b). A significant minority made progress finding the values of *a* and *b*. Many of those candidates reached $a=\frac{5}{4}b$ but were unable to deduce that $\frac{a}{b}=\frac{5}{4}$ and therefore $a=5$ and $b=4$. . The part could be answered correctly by setting the numerator to 0 and making *x* the subject of formula and equating the result to $\frac{5}{4}$. Some candidates were seen setting $a-bx=0$ then using $x=\frac{5}{4}$ to achieve $4x=5$ and rearranged to obtain $4x-5=0$. They then obtained $a=5, \quad b=-4$ by comparing coefficients

Part (c). This part could be done easily setting the value of $x=0$ in the given equation $y=\frac{a-bx}{cx-d}$ and find the value of *y*, but candidates who failed to find the correct values of *a* and *d* in the previous parts could only score the method mark here by reaching $p=\frac{\text{their } a}{-\text{their } d}$ though many candidates did not even attempt this part [or indeed part (d)] of this question.

Part (d). The final B mark was a follow through mark which could be achieved using their values for *b* and *c* here. A small number of candidates achieved $y=-2$ [which was the correct answer] but from an incorrect method. If they did not obtain $y=-2$ by using

$y=\dfrac{\text{their } -b}{\text{their } c}$ this mark could not be awarded.

Question 4

This question was answered well by many candidates achieving full marks.

Part (a) Most candidates realised they needed to differentiate, equate their resulting expression to 0, substitute $x = 3$ in the equation and attempt to find a value for p. Thereafter, use that value of p to find the value of q using the given stationary point. But some candidates did not know how to find the value of p first and instead they found the value of q using the given value of p gaining only one mark in this part of the question. Very few candidates were seen using the first method stated in the main scheme by completing the square.

Part (b). Most candidates differentiated again here and almost all used the second derivative to classify the nature of the stationary point correctly. However, a very small minority of candidates lost the B mark here by giving the conclusion as <u>maximum</u> point despite correctly reaching the second derivative correctly as 4 and stating that it was greater than zero.

A small number of candidates scored the mark by simply stating the coefficient of x^2 is positive, hence the stationary point is a minimum.

Part (c) was generally answered well with many candidates achieving full marks. These are standard questions and students know to substitute $x = 1$ in their differentiated expression to find the gradient of the tangent and to use $m_1 \times m_2 = -1$ m_1 to find the gradient of the normal. The main errors seen in this part were either candidates used the stationary point (3, $(3,-5)$ instead of (1, 3) or -8 instead of $\frac{1}{8}$.

Most students used the slightly more efficient method of $y-y_1=m(x-x_1)$ rather than $y=mx+c$ Most candidates gave the final answer as $x-8y+23=0$ or the equivalent in integer coefficients. A rare answer seen was $\frac{x}{8} - y + \frac{23}{8} = 0$ which was also given full credit as the question did not require a, b and c to be integers.

Question 5

Many candidates scored no marks at all on this question some making no attempt to answer the question at all. This suggests that this topic of small changes may not be familiar. It is covered by 9G in the specification.

Of those who did attempt the question, most got no further than differentiating the product. However, this differentiation was usually correct and well done, including the differentiation of the exponential term to achieve the first two marks only in this question.

Of the minority of candidates who progressed beyond the initial differentiation, the majority were successful in completing the rest of the question and achieving all the remaining marks.

Question 6

Virtually every candidate attempted this question and scored consistently very well.

Part (a) Almost all candidates understood that substitution of $t = 0$ into the given expression for displacement was required. However, a significant proportion of candidates failed to take the modulus of the result and present a positive number for **distance**. This tended to be the only mark that the overwhelming majority of candidates lost throughout this question.

(b) Almost all candidates successfully differentiated the expression given for displacement to find the velocity and then equated this velocity to zero when the particle was at rest. Most of these candidates correctly solved the equation correctly rejected the negative value. The non-rejection of the negative value tended to be the place where just a few candidates lost another mark in this question.

(c) Again, almost all candidates successfully differentiated again and set their acceleration to 10. The relatively small number of candidates who hadn't differentiated correctly in part (b) often moved on to gain 2 out of the 3 marks available for part (c). A few candidates integrated in this part, or else set their expression for the velocity = 10, but these errors were seen only rarely and this straightforward question was the best answered question in the whole paper.

Question 7

Part (a) (i) Virtually every candidates produced two correct equations from the given information, with a roughly equal split seen between the pair of equations possible. Occasionally, candidates confused the sum of three terms with the formula for the third term ar^2. Those choosing to use the formula for the sum, rather than add the first 3 terms ($a + ar + ar^2$), generally tended to be more successful in making further progress, as further manipulation tended to be more straightforward especially when the common factor of $(1-r)$ was cancelled through.

The overwhelming majority of candidates chose to eliminate "a", rather than r; a range of techniques being used such as substitution of a (or $6a$), or $\frac{a}{1-r}$ The most efficient solutions presented involved the two equations being divided to produce an expression for $1 - r^3$, from which r was readily obtained. Many solutions not using this method, often where candidates had used ($a + ar + ar^2$), produced an equation with many terms, most of which cancelled in pairs. A few candidates progressed to a quartic equation and then presented the given value for r. It may be useful to remind candidates that in a "show" question", simply writing down values is insufficient.

Part (a)(ii) Substitution of the given value for r, usually into the formula for the sum to infinity proved straightforward for most – even those who were unable to complete (a) (i). A very small number of candidates gave a decimal approximation for their answer. Several candidates found "a" using "r", and then substituted the value of $\frac{25}{6}$ back into to show r was indeed 4/5.

Part (b) While a significant proportion of candidates were daunted by the requirements, many of those who attempted this part of the question, identified that they needed to express the sum of n terms as being greater than 19.8. In unpicking this inequality to achieve the given result, errors in multiplying out brackets, or in multiplying $\frac{25}{6}$ by a power, were not uncommon. Some were unfamiliar with logs and made errors in the last step. Often, candidates who did work through towards the given result often ended up with the wrong inequality, having not allowed for division by -1 and forgot to reverse the inequality.

Part (c) Although many candidates understood how to take the first step to solve this inequality, the majority of these didn't progress to the correct final answer, either showing a lack of understanding that $\log(p)$ ($p < 1$), leads to division by a negative number and the reversal of the inequality sign again. 13 was the most common answer for those candidates had progressed this far in the question.

Question 8

Most candidates gained 4/9 marks by correctly answering Parts (a) and (b), but Part (c) proved to be challenging for a majority of candidates.

Part (a) Most candidates picked up 2/2 on this section. A minority of candidates lost marks for the following reasons

- Values stated to 1 decimal place
- Values stated as fractions
- Values stated to 3 decimal places or more

Whilst we allow awrt (answers which round to a value) in most questions, correct rounding is required in this type of question, and incorrect rounding or truncating will be penalised.

There were surprisingly a few non-attempts by candidates

Part (b) The vast majority picked up 2/2. A few candidates were seen plotting one or two of the coordinates at the wrong places. Most candidates connected the points with smooth curves but some candidates connected the points partially with curves and partially with straight lines losing the second B mark and a few candidates even connected all the points with straight lines.

Part (c) Many candidates could not access this part of the question at all despite the function being relatively straightforward. Some candidates tried very hard and made multiple attempts, but eventually crossed all attempts and among them, none of the attempts were enough to score any mark. This question differentiates clearly between the ability of candidates. A few candidates used the main scheme method by setting $2x+\frac{3}{x^2}-3=ax+b$ and then found a and b by comparing coefficients.

Most candidates used the ALT method in the mark scheme but there was little progress by the vast majority. Candidates achieving $y = 7 - 2x$ would almost always carry on to plot the line correctly and did show the line intersecting the coordinate axes at (0,7) and (3.5, 0). They would usually pick up the final A mark for the correct roots of the equation. Errors for final A mark would sometimes be answers given to more than 1 decimal place, for example, $x = 0.6$ and $x = 2.35$ were often seen. Rarely were candidates' solutions outside the values of 0.6 and 0.7 and 2.3 and 2.4 respectively. It is important to remind candidates to read the questions carefully before answering and to always check the requirement for rounding.

As usual in this type of question, quite a few candidates just wrote down the roots without any working seen having obtained the roots from their calculators. Suffice it to say, these 'solutions' scored no marks.

Question 9

Most candidates were able to gain early marks on the question in parts (a) and (b) but made little progress beyond that. Cosine Rule was occasionally used throughout, even when dealing with right-angled triangles. Often the last two parts weren't attempted. Candidates who are successful in understanding and solving three-dimensional trigonometry/Pythagoras questions almost invariably draw and label small thumbnail sketches for themselves as they progress through the question.

Part (a) Almost all candidates were able to successfully answer this part of the question, using Pythagoras' Theorem although somewhat disturbingly, $AC = \sqrt{10+10} = \sqrt{20}$ was seen more than once.

Part (b) A large number of candidates were also able to answer this part of the question. The most common method was to halve AC and again use Pythagoras' Theorem for the answer. A smaller number of candidates were able to use the presence of an isosceles triangle to deduce the answer without working for which full credit was given.

Part (c) Of candidates who attempted this part of the question, it was often unclear due to poor notation, which angle candidates were trying to find. There was often a lack of a clear diagram. Even when using the correct triangle, candidates occasionally found the wrong angle.
Triangle *ABD* was commonly used, and as it was a familiar 6, 8, 10 Pythagorean triplet the most common incorrect answer was 36.9 degrees.

Quite a few candidates did not realise that it was necessary to find length *DM* first using Pythagoras theorem or simple trigonometry before attempting to find angle *DMB*.

Part (d) As this part of the question followed on from finding the angle in part (c), attempting to find a different angle in (c) meant there was no proper start for this part of the question. Most candidates who correctly found the angle in (c) attained both marks in (d), although there were a relatively small number of candidates using the incorrect trigonometrical ratio.
Many successful candidates in part (d) used their previously rounded answer of 58.1 and gave an answer of 3.18 cm which we accepted for full marks, even though rounded values should not be used in calculations to find further angles or lengths.

Question 10

Part (a) This was not a particularly well answered question by candidates despite it being a fairly standard question. There was much "fudging" going on and this part of the question tended to score either 0 or 2. Candidates who scored 0 did not clearly show the taking out of the common factor of 9 from the denominator. It was evident that some students did not understand that "show that" means show every step of your working and it was common for candidates to simply write $3\left(1+\frac{x}{3}\right)^{\frac{1}{2}}$ without explicitly showing where the 3 had come from. A few tried to show how to make RHS = LHS but often did not show enough detail for a "show that" question.

Part (b) For the most part this was usually well done. There was a clear understanding of binominal expansion, which has improved over the years. The negative indices did not cause as many issues as expected as candidates appear to be well versed in putting this term in brackets in their expansion. The same however could not be said for the $\left(-\frac{x}{3}\right)$, which caused candidates some issues. The most common error, however, was an incorrect denominator in the fourth term; candidates incorrectly used 3 rather than 3! It was not common but did occasionally happen that a candidate would have a correct un-simplified expansion up to x^3 but then failed to simplify the third and fourth term correctly. The first two terms were usually correct thus gaining the first A mark following a correct method.

Part (c) Many failed to gain the first B mark, which was awarded just for writing down $3\mathrm{f}(x)=(1+2x)\left(1+\frac{x}{3}\right)^{-\frac{1}{2}}$ It was exceptionally common for candidates to multiply "their" f (x) by 3 and thus lose this B mark.

For those candidates who were able to find an expression in (b) the majority were able to gain the first M mark for replacing $\left(1+\frac{x}{3}\right)^{-\frac{1}{2}}$ with their expansion from part (b) and were then able to gain the second method mark for attempting to expand. It would be beneficial to remind candidates about the importance of brackets in this part – very rarely, candidates multiplied their expansion by only $-2x$ due to missing brackets. The accuracy mark for the correct expansion was not always gained in this part partly due to careless simplification from candidates because they failed to collect terms correctly mainly due to incorrect application of negative signs.

Part (d) For candidates who progressed through this question this far the first mark here was for integration of their answer in part (c) which the vast majority were able to do correctly for their expression and so gained the A mark as we awarded the follow through..

The final M mark was twofold; candidates had to substituting the limits the correct way round **and** divide their answers in (c) by 3. This part was attempted in two ways, candidates either divided by 3 at the start and then integrated their expression or to integrate their expression first. However if candidates did not divide by 3 at the start of this part, very few did in a subsequent step – hence losing the 2nd M mark even after correct substitution was seen. If candidates had carried incorrect working forward from part (c) it was common for them to lose the M mark here as many did not show the substitution, with or without a correct answer in part (c) – it is worth reminding candidates once again of the importance of showing substitution.

It was also evident that a small minority of candidates used their calculator to solve this part and thus scored zero marks as the question specifically states "using algebraic integration". Candidates should be reminded that although the calculator is a good checking tool it should not be relied upon to complete questions like these.

Question 11

There were many candidates who only answered Parts (a) and (d)(i). It was unclear whether this was due to running out of time or not having the knowledge required for (b) and (c).

Part (a) This was the best answered part of question 11. The majority of candidates were able to get to the correct equation using a variety of correct methods for the line AB and gain the M and A mark. It is important to note however, some gained the two marks but went onto simplify their equation incorrectly. This did not lose them any further marks in this paper as we ignored any further incorrect processing/simplification of the equation.

(b) This part was problematic for more than half of the candidates. Many not attempting at all to find either gradient in terms of p, resulting in no possibility of marks for this part of the question at all.

Those that did use a correct method were able to find the two correct gradients (for BC and AC) and gain the first three marks. Some then went onto make algebraic errors when they attempted to set the product of their gradients = – 1 and therefore did not get to a 3-term quadratic and only gained the first 4 marks.

A minority of candidates went on to correctly find the product of the two gradients, put equal to – 1 and achieve usually the correct three term quadratic equation which they factorised correctly to get the given value of *p* and hence gain all 7 marks available.

Once again, as in earlier questions, candidates are not showing their methods for solving their 3TQ's one would assume because they are relying on their calculator to do this. As a result of this they may lose the method mark unnecessarily for not showing working to support how they are solving their 3TQ. It would be worth reminding candidates that [even if they do not arrive at the correct 3TQ] they should always be showing a complete method to solve, whether this be by factorising, completing the square or more simply the quadratic formula for quadratics which do not factorise.

Part (c) Again, this part was problematic for most candidates.

There were a variety of methods used here to solve including but not limited to gradients and Pythagoras' – but these did not lead to the correct answer for candidates. A lot of incorrect work was seen and gained no marks; sometimes this was a complete page of work. Candidates should be more sensible here when there are only two makes on offer.

There were of course some correct solutions, but these were few and far between and as a result had an impact on the marks in Part (d)(ii).

Part (d) (i) This part was well answered, almost all could apply the length formula and therefore gained this B mark. It was common to see candidates miss out Parts (b) and (c) and solely attempt Part (d)(i), which is an improvement on previous years where candidates would have missed a simple mark as it is in a larger question at the end of the paper.

Part (d)(ii) There was only a minority of fully correct solutions to part (d). There were many who didn't attempt it at all. (Usually because they didn't find a correct coordinate for *D* in part(c))

For those that did attempt it, there were two successful methods seen. The first, finding the length of *BD* and then using the area of a triangle formula to find the area. Those using this method were usually successful in method, not all gaining the final A mark due to incorrect coordinates for *D*.

The other, most common method was using determinants; this again was successful for those that used the method. There were some errors in the expansion of their determinant, occasionally candidates got terms mixed up losing the second M mark and hence the final A mark. Centres should remind candidate to show their complete method of expansion rather than simply writing down the answer after the calculation.

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